

MODULE HANDBOOK

SEMESTER 2	
Track 2: Computational Mechanics	
Nonlinear FEM	5 ECTS
Boundary element methods	5 ECTS
Stochastic FEM	5 ECTS
Structural optimization	5 ECTS
Computational methods for dynamic system analysis	5 ECTS
Computational biomechanics	5 ECTS
Machine Learning in Computational Mechanics	5 ECTS
Grid generation	5 ECTS

Students must choose 6 courses out of the 8 available.

Module #1	NONLINEAR FEM		
Information	Credit points : 5 ECTS	Workload: 50 hours	Mode: Elective
Instructor(s)	V. Papadopoulos, K. Spiliopoulos		
Content	Issues of continuum mechanics and basic tensor analysis. Introduction to nonlinear analysis. Incremental equations of motion, Green Lagrange strain tensor. Cauchy stress tensor, Piola Kirchhoff stresses, Incremental total and updated Lagrangian formulations. Principle of Virtual work in a non-linear setting. Linearization of non-linear equations of motion and incremental - iterative solution methods. Newton-Raphson algorithm. Path following techniques. Arc-Length. Geometric Non linearity. Finite element method for geometric non – linear problems: Truss and Cable elements, Plane Strain and plane stress elements, Three-dimensional solid elements, Structural elements: beam and general shell elements. Material nonlinearity. Problem statement. Elastoplastic problem in one dimension. Isotropic and Kinematic Hardening. J2 Plasticity. Deviatoric stress. Deviatoric strain. Yield surface. Von Mises & Tresca Yield criteria. Drucker's postulate. Maximum dissipation principle. Associated and non-associated flow rules. Perfect plasticity. Radial return algorithm. Algorithms for isotropic, kinematic, and combined hardening. Algorithmic tangent operator. Finite element method for materially nonlinear problems. Implementation using MSOLVE and Commercial Software.		
Examination	Written final exam. Final grade: 70% examination and 30% exercises & project.		
Requirement for examination	No specific requirement		
Learning outcomes	The knowledge of safeguarding against any kind of failure is very important, as structures nowadays, for better efficiency, are being pushed to be able to operate under extreme loading conditions. On successful completion, graduates will be able to: • have a good understanding of the of inelastic behavior of the continuum structures • have a good understanding of the geometric nonlinear effects on the structures • know the numerical treatment of the geometric and material nonlinearity within the framework of the finite element method		

Module #2	BOUNDARY ELEMENT METHODS		
Information	Credit points: 5 ECTS	Workload: 50h	Mode: Elective
Instructor(s)	G. Tsiatas		
Content	Introduction. Boundary Elements and Finite Elements. Historical development of the BEM. Preliminary Mathematical Concepts. The Gauss-Green theorem. The divergence theorem of Gauss. Green's second identity. The Dirac delta function. The BEM for Potential Problems in Two Dimensions. Fundamental solution. The direct BEM for the Laplace and the Poisson equation. Transformation of the domain integrals to boundary integrals. Numerical Implementation of the BEM. The BEM with constant boundary elements. The Dual Reciprocity Method for Poisson's equation. Computer program for solving the Laplace equation with constant boundary elements. Domains with multiple boundaries. Torsion of non-circular bars. Deflection of elastic membranes. Bending of simply supported plates. Heat transfer problems. Fluid flow problems. The BEM for Two-Dimensional Elastostatic Problems. Equations of plane elasticity. Betti's reciprocal identity. Fundamental solution. Integral representation of the solution. Boundary integral equations. Numerical solution of the boundary integral equations. Body forces. Computer program for solving the plane elastostatic problem with constant boundary elements. Applications. Nonlinear static and dynamic problems using the Analog Equation Method (AEM).		
Examination	Written final exam. Final grade: 50% written examination, 50% exercises.		
Requirement for examination	Delivery of the solved exercises		
Learning outcomes	The course addresses both the researcher and the practicing engineer. On successful completion, students will be able to: • Learn the BEM as a computational method • Solve potential problems described by the Laplace and Poisson equation using the BEM. • Solve linear and nonlinear problems, both static and dynamic, described by second order partial differential equations using the BEM		

Module #3	STOCHASTIC FEM		
Information	Credit points: 5 ECTS	Workload: 50 hours	Mode: Elective
Instructor(s)	V. Papadopoulos		
Content	Uncertainty quantification. The course introduces the principles and methods used to model and analyse uncertainty in structural engineering. It reviews probability theory and statistics, including random variables, probability distributions, moments, covariance, and the basic concepts of stochastic processes. Students study the classification and properties of stochastic processes and random fields, with particular attention to stationarity, ergodicity, spectral representation, and the simulation of stochastic processes and spatially varying material properties and loads. Stochastic differential equations and FEM. The course presents computational approaches for solving stochastic structural problems, including point discretization and spectral methods, the Karhunen–Loève expansion, Monte Carlo simulation, and variance-reduction techniques. It examines how uncertain material properties and loads influence structural response and reliability and solves the corresponding stochastic FEM equations. Machine Learning. The course also explores data-driven modelling, large datasets, and machine-learning algorithms, with applications in structural reliability analysis, Bayesian updating, and digital twins.		
Examination	Written final exam. Final grade: 70% examination and 30% exercises & project.		
Requirement for examination	No specific requirement		
Learning outcomes	The course addresses both the researcher and the practicing engineer. On successful completion, students will be able to: • have an in-depth understanding of the stochastic process theory and simulation methods; • know the mathematical framework and the computational techniques of uncertainty quantification using finite elements; • critically assess the pertinent Codes' requirements, from the point of view of stochastic and reliability analysis of structures		

Module #4	STRUCTURAL OPTIMIZATION		
Information	Credit points: 5 ECTS	Workload: 50 hours	Mode: Elective
Instructor(s)	N. Lagaros, S. Triantafyllou		
Content	Introduction to structural optimization. Definitions formulations and simple examples of structural optimization problems. Optimality criteria – problems with no constraints (unimodal and multimodal functions). Optimality criteria – problems with constraints (Introduction to Lagrange multipliers, KKT conditions). Linear Programming (Introduction, Duality, Simplex Algorithm, examples). Linear Programming (Engineering problems, plastic design of minimum weight). Non-Linear Programming (Gradient methods). Non-Linear Programming (Condition number, diagonal solving, Newton's method). Metaheuristics / Derivative free algorithms. Laboratory class PC (at the PC lab). Formulations of structural optimization problems (Sizing, Shape & Topology optimization). Multi-objective optimization problems & Generic Algorithms. Sizing structural optimization problems. Shape & Topology structural optimization problems.		
Examination	Written final exam. Final grade: 70% examination and 30% exercises & project.		
Requirement for examination	No specific requirement		
Learning outcomes	The course addresses both the researcher and the practicing engineer. On successful completion, students will be able to: • have an in-depth understanding of the problem formulations of the three types of structural optimization problems; • know the mathematical background and the computational implementation of the search algorithms used for solving this type of problems; • critically assess problem formulations obtained from the industry		

Module #5	COMPUTATIONAL METHODS FOR DYNAMIC SYSTEM ANALYSIS		
Information	Credit points: 5 ECTS	Workload: 50h	Mode: Elective
Instructor(s)	A. Chasalevris		
Content	Dynamic models of structures, machines, and mechanisms and methods of derivation of equations of motion (Lagrange's equations). Free and forced vibrations of Single Degree of Freedom (SDOF) systems. Numerical solutions of linear and nonlinear SDOF equation of motion. Damping (viscous, hysteretic) and the role of negative damping in self-excited vibrations in aeroelastic structures, machines, and manufacturing processes. Linearized Multi Degree of Freedom (MDOF) systems, stability indexes and stability assessment with respect to standards. Sources of nonlinearity in structures and machines, and of time-delay in dynamic systems. Excited vibrations of linear MDOF systems using harmonic analysis. Excited vibrations of large nonlinear MDOF systems using collocation methods. Floquet theory and evaluation of periodic limit cycles. Numerical continuation of periodic solutions and evaluation of stable and unstable solution branches. Types of bifurcations, quasi-periodicity, chaos assessment and quality of motion in nonlinear structural and machine dynamics. Applications in turbomachines, aircraft wings, wind-turbines, and machine tools.		
Examination	Written final exam. Final grade: 50% written examination, 50% exercises		
Requirement for examination	Solution of the exercises		
Learning outcomes	The course addresses the role of computational dynamics in the dynamic design of realistic structures and machines. On successful completion, students will be able: • To formulate the dynamic model of a given structure or machine including local nonlinearities • To assess the stability of the system according to standards. • To solve the equations of motion using numerical methods for large nonlinear systems. • To evaluate the quality of motion in periodic, quasi-periodic and chaotic oscillations of structures and machines.		

Module #6	COMPUTATIONAL BIOMECHANICS		
Information	Credit points: 5 ECTS	Workload: 50h	Mode: Elective
Instructor(s)	C. Charitidis, M. Kavousanakis		
Content	Finite Differences, Finite Elements Method: introduction to MATLAB and COMSOL software. Tumor growth models: Discrete/stochastic and continuum level models for the simulation of (a) avascular and (b) vascular tumors. Gene regulatory networks: Network of intracellular reactions for the lac operon network- Cell level deterministic models - Stochastic models: Gillespie algorithm. Biofluid Mechanics: Windkessel model- Navier-Stokes equations - blood flow. Introduction to Biomaterials & Nano-biomaterials- Applications (Tissue Mechanics and Targeted Drug Delivery). Prosthetics and Biomedical applications: 3D Bioprinting - Custom-made implants and Applications. Nanobiomechanics- Mechanical properties - Design of experiments and models for nano-biomaterials. Biomaterials mechanics – Biological response of nanosystems- Cell transport mechanisms and permeability.		
Examination	Written final exam. Final grade: 100% examination.		
Requirement for examination	No specific requirement		
Learning outcomes	On successful completion, graduates will be able to: • Use computational tools for solving problems in biomechanics. • Apply specialized computational techniques and evaluate simulation results by comparing them with available clinical/experimental data. • Integrate engineering knowledge and adapt it to problems in biotechnology and biomedical engineering. • Develop mathematical models of biological processes (e.g., models for the interaction of cancer cells with the nervous and lymphatic systems in solid tumors). • Identify and apply techniques for bacterial and cell culture.		

Module #7	MACHINE LEARNING IN COMPUTATIONAL MECHANICS		
Information	Credit points: 5 ECTS	Workload: 50h	Mode: Elective
Instructor(s)	I. Kalogeris		
Content	Introduction to machine learning in engineering and computational mechanics. Fundamentals of supervised, unsupervised, and reinforcement learning. Regression and classification problems. Data preprocessing, normalization, and feature extraction. Dimensionality reduction. Principal Component Analysis (PCA) and Proper Orthogonal Decomposition (POD). Classical machine learning methods. Linear and nonlinear regression, decision trees, and Support Vector Machines (SVMs). Model training, validation, and performance evaluation. Artificial neural networks and deep learning. Network architectures, training algorithms, regularization, hyperparameter optimization, and generalization. Feedforward (FFNN), Convolutional (CNN), and Recurrent Neural Networks (RNNs). Physics-informed machine learning. Physics-Informed Neural Networks (PINNs). Incorporation of governing differential equations, initial conditions, and boundary conditions into neural network training. Applications to computational mechanics. Neural operators and operator learning. Deep Operator Networks (DeepONets) and Fourier Neural Operators (FNOs). Learning mappings between function spaces for parameterized differential equations. Surrogate modeling and reduced-order modeling of computational mechanics problems. Gaussian Processes, Radial Basis Functions (RBFs), and deep learning surrogates. Uncertainty quantification and active learning. Bayesian learning, predictive uncertainty estimation, and data-efficient modeling. Generative machine learning. Variational Autoencoders (VAEs), Generative Adversarial Networks (GANs), Normalizing Flows, and Diffusion Models. Applications in computational modeling and simulation.		
Examination	Written final examination. Final grade: 50% final exam, 50% numerical programming exercise.		
Requirement for examination	Participation and passing grade in: Numerical programming exercises: One computational project involving the implementation and application of machine learning methods to a computational mechanics problem. Indicative topics include surrogate modeling, reduced-order models, physics-informed neural networks, or neural operators. Implementation using Python-based machine learning frameworks (e.g. Pytorch, Scikit-learn)		

Learning outcomes	The course introduces modern machine learning methodologies and their integration into computational mechanics, bridging data-driven approaches and physics-based numerical modeling. Emphasis is placed on the development and application of machine learning methods for the analysis, modeling, and simulation of engineering problems. On successful completion, students will be able to: • understand the fundamental principles of supervised, unsupervised, and reinforcement learning and identify their potential applications in engineering • preprocess engineering datasets and apply dimensionality reduction techniques, including PCA and POD • implement and evaluate classical machine learning algorithms for regression and classification problems • develop, train, and validate artificial neural networks, including feedforward, convolutional, and recurrent architectures • formulate and apply physics-informed neural networks for solving differential equations arising in computational mechanics • understand the principles of neural operator learning and apply architectures such as DeepONet and FNO to parameterized computational problems • develop surrogate and reduced-order models to approximate the response of computationally expensive numerical simulations • apply uncertainty quantification techniques and understand the role of active learning in data-efficient computational modeling • understand the principles of generative machine learning models and their potential applications in computational mechanics • critically assess the accuracy, generalization capabilities, computational efficiency, and limitations of machine learning approaches in engineering simulation.
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Module #8	GRID GENERATION		
Information	Credit points: 5 ECTS	Workload: 50h	Mode: Elective
Instructor(s)	V. Asouti, K. Giannakoglou		
Content	This course aims at creating a solid theoretical background related to grid generation methods and the way grids can be handled (including their adaptation) in Computational Mechanics, for the solution of problems governed by PDEs. Two- and three-dimensional, structured, and unstructured grid generation methods are discussed. Linking with the way grids are used in CFD and CSM methods. Curvilinear coordinate systems and relevant coordinate transformations. Methods for generating body-fitted coordinate systems using algebraic methods or Laplace or Poisson type PDEs. Grid quality control using appropriate source terms. Unstructured grid topology, their storage and handling, coding tips. Grid generation methods for unstructured grids. Delaunay triangulation and their basic properties. The advancing front method. Three-dimensional unstructured grids with tetrahedral elements or hybrid grids. Adaptation of structured and unstructured grids. Adaptation criteria, refinement and de-refinement schemes. H-type and p-type refinement. Applications using in-house codes for students to gain experience.		
Examination	Written final exam. Final grade: 70% examination and 30% exercises & project.		
Requirement for examination	No specific requirement		
Learning outcomes	On successful completion, graduates will be able to: • have a good understanding of grid types they may use in any kind of simulations in Computational Mechanics, their pros and cons, • have a good understanding of how a physical problem cast in the form of PDEs can be discretized on a structured or unstructured grid, • even if students are not planning to program their own grid generation software, they will become better users of commercial or open-source grid generation software, if this is what they are used to do.		